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The effects of lepton KK modes on the electric dipole moments of the leptons in the Randall–Sundrum scenario

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Abstract. We study the charged lepton electric dipole moments in the Randall–Sundrum model where the leptons and the gauge fields are accessible to the extra dimension. We observe that the electric dipole moment of the electron (muon; tau) reaches a value of the order of $10^{-26}\, e\,\mathrm{cm}$ ($10^{-20}\, e\,\mathrm{cm}$; $10^{-20}\, e\,\mathrm{cm}$) with the inclusion of the lepton KK modes.

1 Introduction

CP violation is among the most interesting physical phenomena, and the electric dipole moments (EDMs) of fermions are important tools to understand it since EDMs are driven by the CP violating interaction. There are various experimental and theoretical works in the literature, and the experimental results of the electron, muon and tau EDMs are $d_e = (1.8 \pm 1.2 \pm 1.0) \times 10^{-27}\, e\,\mathrm{cm}$ [1], $d_\mu = (3.7 \pm 3.4) \times 10^{-19}\, e\,\mathrm{cm}$ [2] and $d_\tau = (3.1) \times 10^{-16}\, e\,\mathrm{cm}$ [3], respectively. Furthermore, the experimental upper bound of the EDM of the neutron has been found to be $d_N < 1.1 \times 10^{-25}\, e\,\mathrm{cm}$ [4, 5]. From the theoretical point of view, the source of CP violation in the standard model (SM) is the complex Cabbibo–Kobayashi–Maskawa (CKM) matrix, the lepton mixing matrix in the quark, and the lepton sector; and the calculation of fermion EDMs shows that their numerical values are negligible in the SM. In [6–9], the EDMs of the quark have been estimated to be $\sim 10^{-30}$ ($e\,\mathrm{cm}$), which is a small quantity since the nonzero contribution exists at least in the three-loop level. In order to enhance the fermion EDMs, one needs an alternative source of CP violation and additional contributions coming from physics beyond the SM. The multi-Higgs doublet models (MHDMs) and the supersymmetric model (SUSY) [10] are among the possible models carrying an additional CP phase. The electron EDM has been predicted to be of the order of the magnitude of $10^{-32}\, e\,\mathrm{cm}$ in the two Higgs doublet model (2HDM), including the tree level flavor changing neutral currents (FCNC) [11] and, in this case, the additional CP sources are new complex Yukawa couplings. The EDMs of fermions have been analyzed in [12, 13], in the 2HDM with the inclusion of non-universal extra dimensions and in the framework of the split fermion scenario. The EDMs of quarks were calculated in the MHDMs, including the 2HDM in [14, 15, 17–22],[1] the fermion EDMs in the SM, with the inclusion of non-commutative geometry, have been estimated in [23], the lepton EDMs have been studied in the seesaw model in [24], the EDMs of nuclei, deutron, neutron and some atoms have been predicted extensively in [25–34], and limits on the dipole moments of leptons have been analyzed in a left–right symmetric model and in E_6 superstring models in [35–37].

In the present work, we analyze the charged lepton EDMs in the framework of the 2HDM, with the inclusion of a single extra dimension, respecting the Randall–Sundrum (RS1) scenario [38, 39]. The extra dimensions are introduced to solve the hierarchy problem between the weak and Planck scales. In the RS1 model, gravity is localized on a 4D boundary, the so called hidden (Planck) brane, and the other fields, including the SM fields, live on another 4D boundary, the so called visible (TeV) brane. The warp factor, which is an exponential function of the compactified radius in the extra dimension, drives the difference of induced metrics on these boundaries. With this factor, two effective scales, the Planck scale M_{Pl} and the weak scale m_{W}, are connected and the hierarchy problem is solved. If the SM fields are accessible to the extra dimension, one obtains a richer phenomenology [40–58]. The fermion mass hierarchy can be obtained by considering that the fermions have different locations in the extra dimension and it is induced by the Dirac mass term in the Lagrangian [44, 47–49]. Reference [50] is devoted to this hierarchy by considering the Higgs field to have an exponential profile around the TeV brane, and [51] is devoted to extensive work on the bulk fields in various multi-brane models. The different locations of the fermion fields in the extra dimension can ensure the flavor violation (FV), and it is carried by the Yukawa interactions, coming from

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[1] For a recent review, see [16].

the SM Higgs–fermion–fermion vertices. The fermion localization in the RS1 background has been applied to the high precision measurements of top pair production at the ILC [56] and the various experimental FCNC constraints and the electroweak precision tests for the location parameters of the fermions in the extra dimension are analyzed in [57, 58].

Here, we study the charged lepton EDMs in the case that the leptons and gauge fields are accessible to the extra dimension with localized leptons in the RS1 background. We observe that d_e (d_μ; d_τ) reach values of the order of $10^{-26}\, e\,\mathrm{cm}$ ($10^{-20}\, e\,\mathrm{cm}$; $10^{-20}\, e\,\mathrm{cm}$) with the inclusion of the KK modes.

The paper is organized as follows: in Sect. 2, we present EDMs of charged leptons in the RS1 scenario, in the 2HDM. Section 3 is devoted to a discussion and to our conclusions. In the appendix, we present a construction of the SM fermions and their KK modes.

2 Electric dipole moments of charged leptons in the two Higgs doublet model, in the RS1 scenario

The fermion EDM arises from the CP violating fermion–fermion–photon effective interaction and, therefore, their experimental and theoretical search ensure considerable information on the nature of CP violation. In the SM, CP violation is driven by the complex CKM matrix elements in the quark sector and a possible lepton mixing matrix in the lepton sector. The tiny theoretical values of fermion EDMs in the SM force one to go beyond present calculations and the extension of the Higgs sector with FCNCs at tree level is one of the possibilities in order to enhance their theoretical values. In the present work, we consider the 2HDM, which allows the FCNC at tree level with the complex Yukawa couplings.[2] The additional effect coming from the extra dimension(s) is another possibility to enhance the CP violation and, here, we consider the RS1 scenario with localized charged leptons in the extra dimension. The RS1 background is curved and the corresponding metric reads

$$\mathrm{d}s^2 = \mathrm{e}^{-2\sigma}\eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu + \mathrm{d}y^2\,, \tag{1}$$

where $\sigma = k|y|$ with the bulk curvature constant k and the exponential $\mathrm{e}^{-k|y|}$, with $y = R|\theta|$, is the warp factor, which drives the hierarchy and rescales all mass terms on the visible brane for $\theta = \pi$. Here R is the compactification radius in the extra dimension that is compactified onto a S^1/Z_2 orbifold with two boundaries, the hidden (Planck) brane and the visible (TeV) brane. In the RS1 model, all SM fields live in the visible brane, however, gravity is accessible to the bulk and it is considered to be localized on the hidden brane. With the assumption that the gauge fields and the fermions also may access the extra dimension, the particle spectrum is extended and the physics becomes richer. In the present work, we consider this scenario with the addition of a Z_2 invariant.[3] The Dirac mass term in the Lagrangian of bulk fermions that results in fermion localization in the extra dimension [41, 43, 44, 46, 47, 49, 50] is

$$\mathcal{S}_m = -\int \mathrm{d}^4x \int \mathrm{d}y\sqrt{-g}\,m(y)\bar{\psi}\psi\,, \tag{2}$$

where $m(y) = m\frac{\sigma'(y)}{k}$ with $\sigma'(y) = \frac{\mathrm{d}\sigma}{\mathrm{d}y}$ and $g = \mathrm{Det}[g_{MN}] = \mathrm{e}^{-8\sigma}$, $M, N = 0, 1, \dots, 4$. Here the bulk fermion is expanded as

$$\psi(x^\mu, y) = \frac{1}{\sqrt{2\pi R}}\sum_{n=0}^{\infty}\psi^{(n)}(x^\mu)\mathrm{e}^{2\sigma}\chi_n(y)\,, \tag{3}$$

with the normalization

$$\frac{1}{2\pi R}\int_{-\pi R}^{\pi R}\mathrm{d}y\,\mathrm{e}^{\sigma}\chi_n(y)\chi_m(y) = \delta_{nm}\,. \tag{4}$$

Using the Dirac equation and the normalization condition, the zero mode fermion is obtained as follows:

$$\chi_0(y) = N_0\mathrm{e}^{-r\sigma}\,, \tag{5}$$

where $r = m/k$ and the normalization constant N_0 reads

$$N_0 = \sqrt{\frac{k\pi R(1-2r)}{\mathrm{e}^{k\pi R(1-2r)} - 1}}\,. \tag{6}$$

On the other hand,

$$\chi_0'(y) = \mathrm{e}^{-\frac{\sigma}{2}}\chi_0(y) \tag{7}$$

is the appropriately normalized solution, and it is localized in the extra dimension. The parameter r is responsible for the localization and for $r > \frac{1}{2}$ ($r < \frac{1}{2}$) it is localized near the hidden (visible) brane.

The action that is responsible for the charged lepton EDMs in the RS1 background is

$$\mathcal{S}_Y = \int \mathrm{d}^5x\sqrt{-g}\left(\xi^E_{5ij}\bar{l}_{iL}\phi_2 E_{jR} + \mathrm{h.c.}\right)\delta(y - \pi R)\,, \tag{8}$$

where L and R denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, ϕ_2 is the new scalar doublet, l_{iL} (E_{jR}) are lepton doublets (singlets), ξ^E_{5ij}, with family indices i, j, are the complex Yukawa couplings in five dimensions, and they induce the FV interactions in the lepton sector. Here, we assume that the Higgs doublet ϕ_1, living on the visible brane, has a non-zero vacuum expectation value to ensure the ordinary masses of the gauge fields and the fermions; however, the

[2] Here, we assume that the CKM type matrix in the lepton sector does not exist, the charged flavor changing (FC) interactions vanish and the lepton FV comes from internal new neutral Higgs bosons, h^0 and A^0.

[3] The fermions have two possible transformation properties under the orbifold Z_2 symmetry, $Z_2\psi = \pm\gamma_5\psi$ and the combination $\bar{\psi}\psi$ is odd. The Z_2 invariant mass term is obtained if the Z_2 odd scalar field is coupled to the combination $\bar{\psi}\psi$.

second doublet, which lies also on the visible brane, has no vacuum expectation value:

$$\phi_1 = \frac{1}{\sqrt{2}}\left[\begin{pmatrix}0\\ v+H^0\end{pmatrix} + \begin{pmatrix}\sqrt{2}\chi^+\\ \mathrm{i}\chi^0\end{pmatrix}\right]\,,$$
$$\phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2}H^+\\ H_1+\mathrm{i}H_2\end{pmatrix}\,, \tag{9}$$

and

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}\,;\quad \langle\phi_2\rangle = 0\,. \tag{10}$$

The gauge and CP invariant Higgs potential that spontaneously breaks $\mathrm{SU}(2)\times\mathrm{U}(1)$ down to $\mathrm{U}(1)$ reads

$$\begin{aligned}V(\phi_1,\phi_2) &= c_1\left(\phi_1^+\phi_1 - v^2/2\right)^2 + c_2\left(\phi_2^+\phi_2\right)^2\\ &+ c_3\left[\left(\phi_1^+\phi_1 - v^2/2\right) + \phi_2^+\phi_2\right]^2\\ &+ c_4\left[\left(\phi_1^+\phi_1\right)\left(\phi_2^+\phi_2\right) - \left(\phi_1^+\phi_2\right)\left(\phi_2^+\phi_1\right)\right]\\ &+ c_5\left[\mathrm{Re}\left(\phi_1^+\phi_2\right)\right]^2 + c_6\left[\mathrm{Im}\left(\phi_1^+\phi_2\right)\right]^2 + c_7\,,\end{aligned} \tag{11}$$

with constants c_i, $i=1,\ldots,7$. The choice (10) and the potential (11) leads to the fact that the SM particles are collected in the first doublet and the new particles in the second one. This is the case that no mixing occurs between CP even neutral Higgs bosons H^0 and h^0 in the tree level and H_1 and H_2 in (9) are obtained as the mass eigenstates h^0 and A^0, respectively.

The lepton doublet and singlet fields in (8) are expanded to their KK modes by

$$\begin{aligned}l_{iL}(x^\mu,y) &= \frac{1}{\sqrt{2\pi R}}\mathrm{e}^{2\sigma} l_{iL}^{(0)}(x^\mu)\chi_{iL0}(y)\\ &+ \frac{1}{\sqrt{2\pi R}}\sum_{n=1}^{\infty}\mathrm{e}^{2\sigma}\left(l_{iL}^{(n)}(x^\mu)\chi_{iLn}^l(y) + l_{iR}^{(n)}(x^\mu)\chi_{iRn}^l(y)\right)\,,\\ E_{jR}(x^\mu,y) &= \frac{1}{\sqrt{2\pi R}}\mathrm{e}^{2\sigma}E_{jR}^{(0)}(x^\mu)\chi_{jR0}(y)\\ &+ \frac{1}{\sqrt{2\pi R}}\sum_{n=0}^{\infty}\mathrm{e}^{2\sigma}\left(E_{jR}^{(n)}(x^\mu)\chi_{jRn}^E(y) + E_{jL}^{(n)}(x^\mu)\chi_{jLn}^E(y)\right)\,,\end{aligned} \tag{12}$$

where the zero mode leptons $\chi_{iL0}(y)$ and $\chi_{jR0}(y)$ are given in (5) with the replacements $r\to r_{iL}$ and $r\to r_{iR}$, respectively. For the effective Yukawa coupling ξ_{ij}^E, we integrate out the Yukawa interaction (8) over the fifth dimension by taking the zero mode lepton doublets, singlets, and neutral Higgs fields $S=h^0, A^0$:

$$\begin{aligned}V_{RLij}^0 &= \frac{\xi_{5ij}^E}{2\pi R}\int_{-\pi R}^{\pi R}\mathrm{d}y\,\chi_{iR0}(y)\chi_{jL0}(y)\delta(y-\pi R)\\ &= \frac{\mathrm{e}^{-k\pi R(r_{iR}+r_{jL})}k\sqrt{(1-2r_{iR})(1-2r_{jL})}}{\sqrt{(\mathrm{e}^{k\pi R(1-2r_{iR})}-1)(\mathrm{e}^{k\pi R(1-2r_{jL})}-1)}}\xi_{5ij}^E\,.\end{aligned} \tag{13}$$

Here, we embed the vertex factor $V_{RL(LR)ij}^0$ into the coupling $\xi_{ij}^E\left((\xi_{ij}^E)^\dagger\right)$ and fix the numerical value of $\xi_{ij}^E\left((\xi_{ij}^E)^\dagger\right)$ by assuming that the coupling ξ_{5ij}^E in five dimensions is flavor dependent. In this case the hierarchy of the new Yukawa couplings is not related to the lepton field locations. On the other hand, we need the vertex factors due to S–KK mode charged lepton–charged lepton couplings $V_{LR(RL)ij}^n$ since charged lepton KK modes exist in the internal line (see Fig. 1). After the integration of the Yukawa interaction (8) over the fifth dimension, the S-zero mode lepton singlet (doublet)–KK mode lepton doublet (singlet) vertex factor (see the appendix for the construction of KK mode charged lepton doublets and singlets) reads

$$\begin{aligned}V_{RL(LR)ij}^n = \frac{\xi_{5ij}^E\left((\xi_{5ij}^E)^\dagger\right)}{2\pi R}\int_{-\pi R}^{\pi R}\mathrm{d}y\,\chi_{iR0(iL0)}(y)\chi_{jLn(jRn)}(y)\\ \times\delta(y-\pi R)\,,\end{aligned} \tag{14}$$

and

$$\begin{aligned}V_{RLij}^n &= N_{Ln}\mathrm{e}^{k\pi R(1/2-r_{iR})}\\ &\times\left(J_{\frac{1}{2}-r_{jL}}\left(\mathrm{e}^{k\pi R}x_{nL}\right) + c_L Y_{\frac{1}{2}-r_{jL}}(\mathrm{e}^{k\pi R}x_{nL})\right)\\ &\times\frac{1}{\pi R\sqrt{\frac{\mathrm{e}^{k\pi R(1-2r_{iR})}-1}{k\pi R(1-2r_{iR})}}}\xi_{5ij}^E\,,\\ V_{LRij}^n &= N_{Rn}\mathrm{e}^{k\pi R(1/2-r_{iL})}\\ &\times\left(J_{\frac{1}{2}+r_{jR}}(\mathrm{e}^{k\pi R}x_{nR}) + c_R Y_{\frac{1}{2}+r_{jR}}(\mathrm{e}^{k\pi R}x_{nR})\right)\\ &\times\frac{1}{\pi R\sqrt{\frac{\mathrm{e}^{k\pi R(1-2r_{iL})}-1}{k\pi R(1-2r_{iL})}}}(\xi_{5ij}^E)^\dagger\,,\end{aligned}$$

where c_L (c_R) is given in (A.4) (A.7) with the replacements $r\to r_{jL}$ $(r\to r_{jR})$. Here the effective S-zero mode lepton singlet (doublet)–KK mode lepton doublet (singlet) coupling $\xi_{ij}^{En}(\xi_{ij}^{En})^\dagger$ reads

$$\xi_{ij}^{En}\left((\xi_{ij}^{En})^\dagger\right) = \frac{V_{RL(LR)ij}^n}{V_{RL(LR)ij}^0}\xi_{ij}^E\,. \tag{15}$$

Notice that the strengths of the S–KK mode charged lepton–charged lepton couplings are regulated by the locations of the lepton fields and we use two different sets, set I and set II.[4]

The effective EDM interaction for a fermion f reads

$$\mathcal{L}_{\mathrm{EDM}} = \mathrm{i}d_f\bar{f}\gamma_5\sigma^{\mu\nu}fF_{\mu\nu}\,, \tag{16}$$

where $F_{\mu\nu}$ is the electromagnetic field tensor, 'd_f' is the EDM of the fermion; it is a real number by hermiticity (see Fig. 1 for the one-loop diagrams that contribute to the EDMs of fermions).

[4] In this scenario, the source of FV is not related to the different locations of the fermion fields in the extra dimension [54, 55], but it is carried by the new Yukawa couplings in four dimensions and the additional effect due to the extra dimension is the enhancement in the physical quantities of the processes studied.

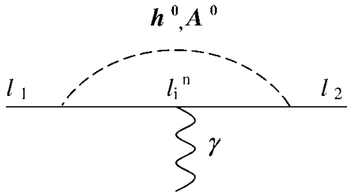

Fig. 1. One-loop diagrams contributing to EDM of l-lepton due to the neutral Higgs bosons h^0 and A^0 in the 2HDM. The *wavy (dashed-solid) line* represents the electromagnetic field (h^0 or A^0 fields, charged lepton fields and their KK modes)

Now, we present the charged lepton EDMs with the addition of KK modes in the framework of the RS1 scenario. Since there is no CKM type lepton mixing matrix according to our assumption, only the neutral Higgs part gives a contribution to their EDMs, and the EDM of lepton l 'd_l' ($l = e, \mu, \tau$) can be calculated as a sum of contributions coming from the neutral Higgs bosons h_0 and A_0. For $l = e, \mu$ we get[5]

$$d_l = -\frac{\mathrm{i}G_{\mathrm{F}}}{\sqrt{2}}\frac{e}{32\pi^2}Q_\tau\Bigg\{\frac{1}{m_\tau}\left(\left(\bar{\xi}^{E*}_{\mathrm{N},l\tau}\right)^2-\left(\bar{\xi}^{E}_{\mathrm{N},\tau l}\right)^2\right) \times\left(\left(F_1(y_{h^0})-F_1(y_{A_0})\right)\right)+\sum_{n=1}^{\infty}\left(\left(\bar{\xi}^{En*}_{\mathrm{N},l\tau}\right)^2-\left(\bar{\xi}^{En}_{\mathrm{N},\tau l}\right)^2\right) \times\left(G(y_{nL,h^0},y_{nR,h^0})-G\left(y_{nL,A^0},y_{nR,A^0}\right)\right)\Bigg\}, \quad (17)$$

where

$$G(y_{nL,S},y_{nR,S}) = G_1(y_{nL,S},y_{nR,S})+G_1(y_{nR,S},y_{nL,S}) + G_2(y_{nL,S},y_{nR,S})+G_2(y_{nR,S},y_{nL,S})\,, \quad (18)$$

with

$$F_1(w) = \frac{w(3-4w+w^2+2lnw)}{(-1+w)^3}\,, \quad (19)$$

and

$$\begin{aligned}
G_1(y_{nL,S},y_{nR,S}) &= y_{nR,S}\left[m_{nL}(y_{nR,S}-1)y_{nR,S} - m_{nR}(y_{nL,S}(y_{nR,S}-2)+y_{nR,S})\right] \\
&\quad\times\frac{1}{m_S^2(y_{nR,S}-y_{nL,S})^2(1-y_{nR,S})^2}\ln y_{nR,S}\,, \\
G_1(y_{nR,S},y_{nL,S}) &= G_1(y_{nL,S},y_{nR,S})|_{y_{nL,S}\Leftrightarrow y_{nR,S};m_{nL}\Leftrightarrow m_{nR}}\,, \\
G_2(y_{nL,S},y_{nR,S}) &= \frac{y_{nL,S}m_{nL}}{m_S^2(y_{nR,S}-y_{nL,S})(1-y_{nL,S})}\,, \\
G_2(y_{nR,S},y_{nL,S}) &= G_2(y_{nL,S},y_{nR,S})|_{y_{nL,S}\Leftrightarrow y_{nR,S};m_{nL}\Leftrightarrow m_{nR}}\,. \quad (20)
\end{aligned}$$

[5] In the following we use the dimensionful coupling $\bar{\xi}^{E}_{\mathrm{N},ij}$ in four dimensions, with the definition $\xi^{E}_{\mathrm{N},ij} = \sqrt{\frac{4G_{\mathrm{F}}}{\sqrt{2}}}\,\bar{\xi}^{E}_{\mathrm{N},ij}$ where N denotes the word "neutral".

The EDM of the tau lepton reads

$$\begin{aligned}
d_\tau = &-\frac{\mathrm{i}G_{\mathrm{F}}}{\sqrt{2}}\frac{e}{16\pi^2}Q_\tau\Bigg\{\frac{1}{m_\tau}\left(\left(\bar{\xi}^{E*}_{\mathrm{N},\tau\tau}\right)^2-\left(\bar{\xi}^{E}_{\mathrm{N},\tau\tau}\right)^2\right) \\
&\times\int_0^1\mathrm{d}x\int_0^{1-x}\mathrm{d}y(x-1)\left(\frac{y_{h^0}}{L_{h^0}}-\frac{y_{A^0}}{L_{A^0}}\right) \\
&+\sum_{n=1}^{\infty}\int_0^1\mathrm{d}x\int_0^{1-x}\mathrm{d}y \\
&\times\Bigg(\left(\bar{\xi}^{En}_{\mathrm{N},\tau\tau}\right)^2(m_{nR}y+m_{nL}(1-x-y)) \\
&\times\left(\frac{1}{m_{h^0}^2L_{n,h^0}}-\frac{1}{m_{A^0}^2L_{n,A^0}}\right) \\
&-\left(\bar{\xi}^{En*}_{\mathrm{N},\tau\tau}\right)^2(m_{nL}y+m_{nR}(1-x-y)) \\
&\times\left(\frac{1}{m_{h^0}^2L'_{n,h^0}}-\frac{1}{m_{A^0}^2L'_{n,A^0}}\right)\Bigg)\Bigg\}\,, \quad (21)
\end{aligned}$$

where

$$\begin{aligned}
L_S &= x+(x-1)^2y_S\,, \\
L_{n,S} &= x+x(x-1)y_S+yy_{nR,S}+(1-x-y)y_{nL,S}\,, \\
L'_{n,S} &= L_{n,S}|_{y_{nL,S}\Leftrightarrow y_{nR,S}}\,. \quad (22)
\end{aligned}$$

Here $y_S = \frac{m_\tau^2}{m_S^2}$, $y_{nL(nR),S} = \frac{m_{nL(nR)}^2}{m_S^2}$ and Q_τ is the tau lepton charge. In (17) and (21) we take into account only the internal τ-lepton contribution, respecting our assumption that the Yukawa couplings $\bar{\xi}^{E}_{\mathrm{N},ij}, i, j = e, \mu$, are small compared to $\bar{\xi}^{E}_{\mathrm{N},\tau i} i = e, \mu, \tau$ (see Sect. 3 for details). Notice that we make our calculations in arbitrary photon four momentum square q^2 and take $q^2 = 0$ at the end. For the Yukawa couplings we used the parametrization

$$\bar{\xi}^{En}_{\mathrm{N},\tau l} = \left|\bar{\xi}^{En}_{\mathrm{N},\tau l}\right|\mathrm{e}^{\mathrm{i}\theta_l}\,, \quad (23)$$

where $l = e, \mu, \tau$. Here, θ_l is the CP violating parameter, which is the source of the EDM of the lepton.[6]

3 Discussion

In this section, we study the effects of lepton KK modes on the EDMs of charged leptons in the framework of the RS1 scenario, with extended Higgs sector. The source of fermion EDMs is the CP violating interaction that arises from a CP violating phase. Here, we assume that this phase comes from the complex Yukawa couplings appearing in the tree level fermion–fermion–new Higgs interactions, in the framework of the 2HDM. In the case of charged leptons, the leptonic complex Yukawa couplings

[6] The Yukawa factors in (17) can be written as

$$\left(\left(\bar{\xi}^{En*}_{\mathrm{N},l\tau}\right)^2-\left(\bar{\xi}^{En}_{\mathrm{N},\tau l}\right)^2\right) = -2\mathrm{i}\sin 2\theta_l\left|\bar{\xi}^{En}_{\mathrm{N},\tau l}\right|^2\,. \quad (24)$$

$\bar{\xi}^{E}_{N,ij}$, $i,j=e,\mu,\tau$, are responsible for the EDMs and they are free parameters in the model considered. Here, we expect that the Yukawa couplings $\bar{\xi}^{E}_{N,ij}$, $i,j=e,\mu$, are weak compared to $\bar{\xi}^{E}_{N,\tau i}$, $i=e,\mu,\tau$, and the couplings $\bar{\xi}^{E}_{N,ij}$ in four dimensions are symmetric with respect in the indices i and j. Finally, we consider the coupling $\bar{\xi}^{E}_{N,\tau e}$, ($\bar{\xi}^{E}_{N,\tau\mu}$, $\bar{\xi}^{E}_{N,\tau\tau}$) to be dominant among the others. This is the case that the tau lepton and its KK mode appear in the internal line (see Fig. 1). The numerical value of the coupling $\bar{\xi}^{E}_{N,\tau\mu}$ is chosen by respecting the experimental uncertainty, 10^{-9}, in the measurement of the muon anomalous magnetic moment [59] (see [60] for details).[7] This upper limit and the experimental upper bound of BR of $\mu\to e\gamma$ decay, $\mathrm{BR}\le 1.2\times10^{-11}$, can give clues about the numerical value of the coupling $\bar{\xi}^{E}_{N,\tau e}$ [11], and we take it to be of the order of 10^{-2} (GeV). For the coupling $\bar{\xi}^{E}_{N,\tau\tau}$ there is no stringent prediction and we consider an intermediate value that is greater than the coupling $\bar{\xi}^{E}_{N,\tau\mu}$. For the CP violating parameter that drives the EDM interaction we choose the range, $0.1\ge\sin\theta_{e(\mu,\tau)}\ge 0.7$.

In the present work, we study the charged lepton EDMs in the RS1 background with the assumption that the leptons may also access the extra dimension. The inclusion of extra dimensions brings about additional contributions that come from the KK modes of leptons in the 4D effective theory after compactification. Here, we assume that the lepton fields are localized in the extra dimension with the help of the Dirac mass term $m_l=r\sigma'$, $\sigma=k|y|$, see (2), in the action. This is the case that the SM leptons, the right and left handed parts, are located in the extra dimension with exponential profiles (see (5)), which makes it possible to explain the different flavor mass hierarchies (see the appendix for the construction of the SM fields and their masses).

The gauge sector of the model should necessarily live in the extra dimension if the leptons exist in the bulk and, therefore, their KK modes appear after compactification of the extra dimension. These KK modes result in additional FCNC effects at tree level coming from the couplings with charged leptons and they should be suppressed even for low KK masses, by choosing the lepton location parameters c_L (c_R) appropriately [57, 58]. Here, we use two different sets of locations of charged leptons (Table 1) in order to obtain the masses of different flavors, and we verify the various experimental FCNC constraints with KK neutral gauge boson masses as low as few TeVs. In both sets, we estimate the right handed locations of the leptons by choosing the left handed charged lepton locations to be the same. For the case that the left handed charged lepton locations are near the visible brane (see set II), the strengths of the couplings of leptons with the new Higgs doublet living in the 4D brane become stronger and, therefore, the physical quantities related to these couplings are enhanced.

Throughout our calculations we use the input values given in Table 2. Furthermore, the curvature parameter k and the compactification radius R are the additional free parameters of the theory. Here we take $kR=10.83$ and consider the region $10^{17}\le k\le 10^{18}$ (see the discussion in the appendix and in [49]).

In Fig. 2, we present the parameter k dependence of the electron EDM d_e, for $\bar{\xi}^{E}_{N,\tau e}=0.01$ (GeV) and two different values of the CP violating parameter $\sin\theta$. Here, the lower–upper solid (dashed, small dashed) line represents the d_e for $\sin\theta=0.1$–0.5 without KK modes (with KK modes set I, II). It is observed that the d_e is of the order of 10^{-29}–10^{-28} $e\,$cm for $\sin\theta=0.1$–0.5 without KK modes. The inclusion of the KK modes enhances the d_e 50 times-two orders for set I–II for the values of the curvature scale $k\sim10^{17}$ (GeV) and this enhancement becomes weak for $k\sim10^{18}$ (GeV). For set II, the enhancement in d_e is almost two times larger than the case of set I. This is

[7] In [60], the upper limit of the coupling $\bar{\xi}^{E}_{N,\tau\mu}$ is estimated to be ~ 30 (GeV) in the framework of the 2HDM, and here, we take a numerical value that is less than this quantity.

Table 1. Two possible locations of charged lepton fields. Here r_L and r_R are left handed and right handed lepton field location parameters, respectively

	set I		set II	
	r_L	r_R	r_L	r_R
e	–0.4900	0.8800	–1.0000	0.8860
μ	–0.4900	0.7160	–1.0000	0.7230
τ	–0.4900	0.6249	–1.0000	0.6316

Table 2. The values of the input parameters used in the numerical calculations

Parameter	Value
m_μ	0.106 (GeV)
m_τ	1.78 (GeV)
m_{h^0}	100 (GeV)
m_{A^0}	200 (GeV)
G_F	1.16637×10^{-5} (GeV^{-2})

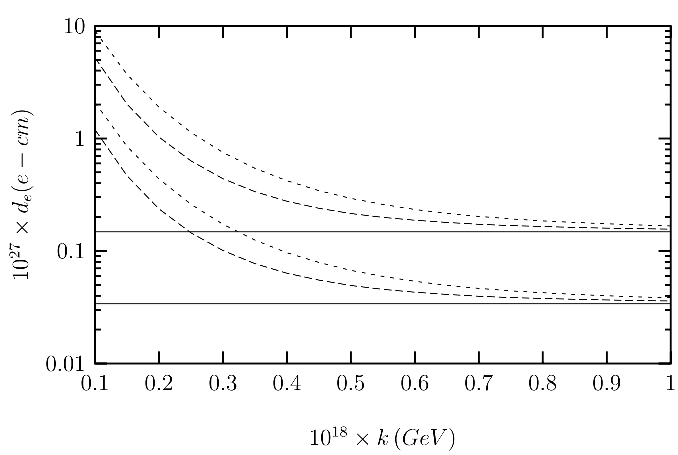

Fig. 2. The dependence of the electron EDM d_e on the parameter k, for $\bar{\xi}^{E}_{N,\tau e}=0.01$ (GeV). Here, the *lower-upper solid* (*dashed, small dashed*) *line* represents d_e for $\sin\theta=0.1$–0.5 without KK modes (with KK modes set I, II)

due to the fact that the left handed leptons (zero and KK modes) are near the visible brane and their couplings to the new Higgs scalars become stronger for the case of set II. The experimental upper limit is $d_e = (1.8 \pm 1.2 \pm 1.0) \times 10^{-27}\, e\,\mathrm{cm}$ and this numerical value can be reached even for small values of the CP parameter $\sin\theta \sim 0.1$ with the inclusion of charged lepton KK modes.

Figure 3 is devoted to the dependence on the parameter k of d_μ, for $\bar{\xi}^E_{N,\tau\mu} = 1$ (GeV) and for two different values of the CP violating parameter $\sin\theta$. Here, the lower–upper solid (dashed, small dashed) line represents d_μ for $\sin\theta = 0.1$–0.5 without KK modes (with KK modes set I, II). We observe that d_μ is of the order of 10^{-25} – $10^{-24}\, e\,\mathrm{cm}$ for $\sin\theta = 0.1$–0.5 without KK modes. With the inclusion of the KK modes d_μ increases to values of the order of $10^{-23}; 2.0 \times 10^{-23}\, e\,\mathrm{cm}$ for $\sin\theta = 0.1$ and $5.0 \times 10^{-23}; 10^{-22}\, e\,\mathrm{cm}$ for $\sin\theta = 0.5$ in the case of set I and set II, for values of the curvature scale $k \sim 10^{17}$ (GeV). With the choice of $\bar{\xi}^E_{N,\tau\mu} = 10$ (GeV), which is the numerical value near the upper limit that is obtained by respecting the experimental uncertainty, 10^{-9}, in the measurement of the anomalous magnetic moment of the muon [60], d_μ reaches a value of $10^{-20}\, e\,\mathrm{cm}$ for $\sin\theta \sim 0.5$, with the inclusion of charged lepton KK modes in the case of set II and $k \sim 10^{17}$ (GeV). This is a numerical value close to the experimental upper limit $d_\mu = (3.7 \pm 3.4) \times 10^{-19}\, e\,\mathrm{cm}$.

Figure 4 represents the parameter k dependence of d_τ, for $\bar{\xi}^E_{N,\tau\tau} = 10$ (GeV) and two different values of the CP violating parameter $\sin\theta$. Here, the lower–upper solid (dashed, small dashed) line represents the d_τ for $\sin\theta = 0.1$–0.5 without KK modes (with KK modes set I and II). Here it is observed that d_τ is of the order of 10^{-23}–$10^{-22}\, e\,\mathrm{cm}$ for $\sin\theta = 0.1$–0.5 without KK modes. The inclusion of the KK modes causes d_τ to be enhanced to values of the order of $10^{-21}; 2.0 \times 10^{-21}\, e\,\mathrm{cm}$ for $\sin\theta = 0.1$ and $5.0 \times 10^{-21}; 10^{-20}\, e\,\mathrm{cm}$ for $\sin\theta = 0.5$ in the case of set I and II, for values of the curvature scale $k \sim 10^{17}$ (GeV). The experimental upper limit of d_τ, $|d_\tau| < (3.1) \times 10^{-16}\, e\,\mathrm{cm}$ is almost two orders away from the theoretical value obtained, even with the strong coupling $\bar{\xi}^E_{N,\tau\tau} \sim 100$ (GeV), and it needs more sensitive experimental measurements.

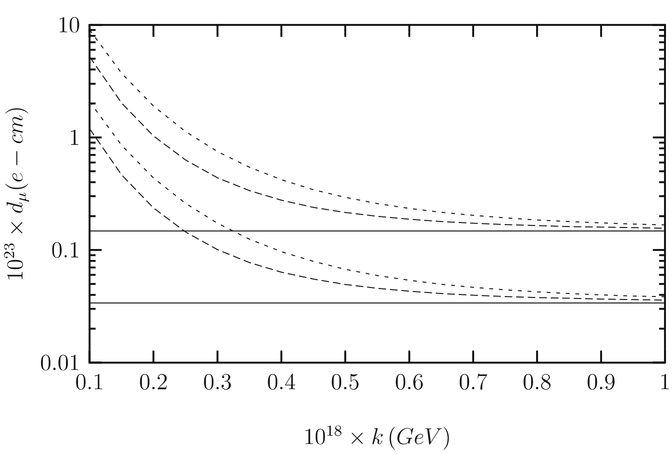

Fig. 3. The same as Fig. 2, but for d_μ and $\bar{\xi}^E_{N,\tau\mu} = 1$ (GeV)

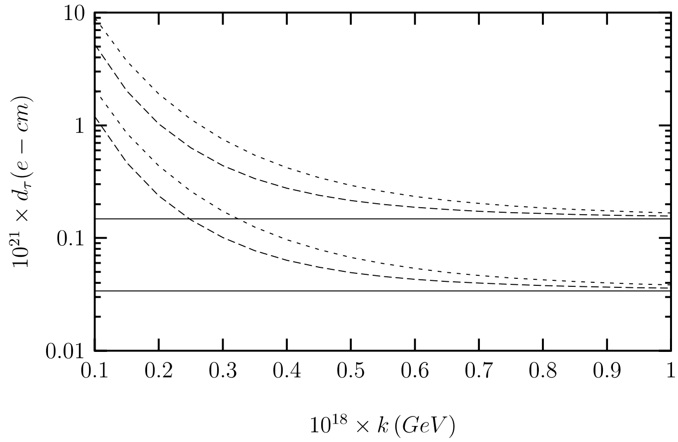

Fig. 4. The same as Fig. 2, but for d_τ and $\bar{\xi}^E_{N,\tau\tau} = 10$ (GeV)

Now, we analyze the dependence on the CP violating parameter $\sin\theta$ of the charged lepton EDMs, for completeness.

In Fig. 5, we present the parameter $\sin\theta$ dependence of d_e; d_μ; d_τ, for $\bar{\xi}^E_{N,\tau e} = 0.01$ (GeV); $\bar{\xi}^E_{N,\tau\mu} = 1.0$ (GeV); $\bar{\xi}^E_{N,\tau\tau} = 10$ (GeV) and for $k = 10^{18}$ (GeV). Here, the lower–intermediate–upper solid (dashed, small dashed) line represents the d_e–d_μ–d_τ without KK modes (with KK modes set I and II). Here it is observed that the EDMs are not so sensitive to the location of lepton fields in the bulk for large values of the curvature parameter k, $k \sim 10^{18}$ (GeV) and the enhancements in the EDMs are %6(%12) for set I (set II). This sensitivity becomes weak for a small value of the CP violating parameter $\sin\theta$.

With more accurate experimental investigations of the charged lepton EDMs it will be possible to understand the mechanism behind the CP violation and one will get

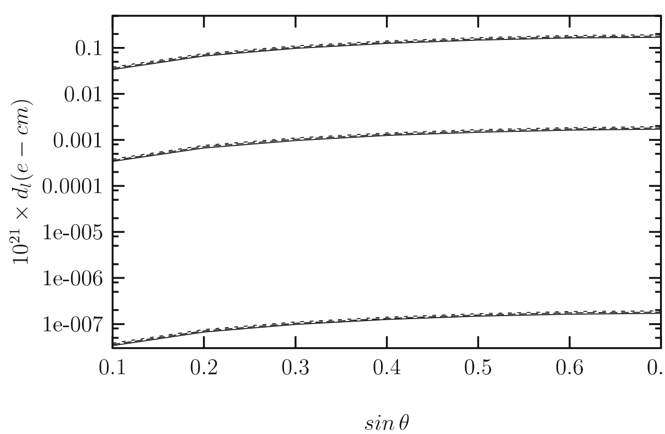

Fig. 5. The dependence of d_e; d_μ; d_τ, on the parameter $\sin\theta$ for $\bar{\xi}^E_{N,\tau e} = 0.01$ (GeV); $\bar{\xi}^E_{N,\tau\mu} = 1.0$ (GeV); $\bar{\xi}^E_{N,\tau\tau} = 10$ (GeV) and for $k = 10^{18}$ (GeV). Here, the *lower–intermediate–upper solid (dashed, small dashed) line* represents d_e–d_μ–d_τ without KK modes (with KK modes set I, II)

powerful information on the effects of warped extra dimensions, if they exist.

Appendix :

The SM fermions are constructed by considering the $SU(2)_L$ doublet ψ_L and the singlet ψ_R, satisfying two separate Z_2 projection conditions: $Z_2\psi_R=\gamma_5\psi_R$ and $Z_2\psi_L=-\gamma_5\psi_L$ (see for example [41]). The zero mode fermions can get mass through the Z_2 invariant left handed fermion–right handed fermion–Higgs interaction, $\bar{\psi}_R\psi_L H$[8] and one gets the location parameters of the left and the right handed parts of the fermions and obtains the current masses of fermions of different flavors. If we consider the SM Higgs field to live on the visible brane, the masses of fermions are calculated by using the integral

$$m_i=\frac{1}{2\pi R}\int_{-\pi R}^{\pi R} \mathrm{d}y \lambda_5 \chi_{iL0}(y)\chi_{iR0}(y)\langle H\rangle \delta(y-\pi R)\,, \tag{A.1}$$

where λ_5 is the coupling in five dimensions, and it can be parametrized in terms of the one in four dimensions, the dimensionless coupling λ: $\lambda_5=\lambda/\sqrt{k}$. Here the expectation value of the Higgs field $\langle H\rangle$ reads $\langle H\rangle = v/\sqrt{k}$, where v is the vacuum expectation value, $v=0.043 M_{\mathrm{Pl}}$, in order to provide the measured gauge boson masses [49] and choose $kR=10.83$ in order to get the correct effective scale on the visible brane, i.e., $M_{\mathrm{W}}=\mathrm{e}^{-\pi kR}M_{\mathrm{Pl}}$ is of the order of TeV.

Since the EDMs of fermions exist at least in the one-loop level, there appear the S-charged lepton–KK charged lepton vertices. The Z_2 projection condition $Z_2\psi=-\gamma_5\psi$ are used to construct the left handed fields on the branes, and as a result the left handed zero and KK modes appear, and the right handed KK modes disappear on the branes. Here the boundary conditions, coming from the Dirac mass term in the action (2), are

$$\left(\frac{\mathrm{d}}{\mathrm{d}y}-m\right)\chi^l_{iLn}(y_0)=0\,, \qquad \chi^l_{iRn}(y_0)=0\,, \tag{A.2}$$

where $y_0=0$ or πR. The left handed lepton $\chi^l_{iLn}(y)$ that lives on the visible brane,

$$\chi^l_{iLn}(y)=N_{Ln}\mathrm{e}^{\sigma/2}\left(J_{\frac{1}{2}-r}\left(\mathrm{e}^{\sigma}x_{nL}\right)+c_L Y_{\frac{1}{2}-r}\left(\mathrm{e}^{\sigma}x_{nL}\right)\right)\,, \tag{A.3}$$

is obtained by using the Dirac equation for the KK mode leptons. Here the constant c_L is

$$c_L=-\frac{J_{-r-\frac{1}{2}}(x_{nL})}{Y_{-r-\frac{1}{2}}(x_{nL})}\,, \tag{A.4}$$

[8] Here, we consider different location parameters r for each left handed and right handed part of different flavors.

where N_{Ln} is the normalization constant and $x_{nL}=\frac{m_{Ln}}{k}$. The functions $J_\beta(w)$ and $Y_\beta(w)$ appearing in (A.3) are the Bessel function of the first kind and of the second kind, respectively. The right handed zero mode fields on the branes can be constructed by considering the Z_2 projection condition $Z_2\psi=\gamma_5\psi$ and this ensures that the right handed zero mode appears, and the right (left) handed KK modes appear (disappear) on the branes with the boundary conditions

$$\left(\frac{\mathrm{d}}{\mathrm{d}y}+m\right)\chi^E_{iRn}(y_0)=0\,, \qquad \chi^E_{iLn}(y_0)=0\,. \tag{A.5}$$

Similarly, the right handed lepton $\chi^E_{iRn}(y)$, which lives on the visible brane, is calculated to be

$$\chi^E_{iRn}(y)=N_{Rn}\mathrm{e}^{\sigma/2}\left(J_{\frac{1}{2}+r}\left(\mathrm{e}^{\sigma}x_{nR}\right)+c_R Y_{\frac{1}{2}+r}\left(\mathrm{e}^{\sigma}x_{nR}\right)\right)\,, \tag{A.6}$$

by using the Dirac equation for the KK mode leptons. Here c_R reads

$$c_R=-\frac{J_{r-\frac{1}{2}}(x_{nR})}{Y_{r-\frac{1}{2}}(x_{nR})}\,, \tag{A.7}$$

where N_{Rn} is the normalization constant and $x_{nR}=\frac{m_{Rn}}{k}$. Notice that the constant c_L, the nth KK mode mass m_{Ln} in (A.3) and the constant c_R, the nth KK mode mass m_{Rn} in (A.6), are obtained by using the boundary conditions (A.2) and (A.5), respectively. For $m_{L(R)n}\ll k$ and $kR\gg 1$ they are approximated as

$$\begin{aligned} m_{Ln} &\simeq k\pi\left(n-\frac{\frac{1}{2}-r}{2}+\frac{1}{4}\right)\mathrm{e}^{-\pi kR}\,,\\ m_{Rn} &\simeq k\pi\left(n-\frac{\frac{1}{2}+r}{2}+\frac{1}{4}\right)\mathrm{e}^{-\pi kR} \quad \text{for } r<0.5\,,\\ m_{Rn} &\simeq k\pi\left(n+\frac{\frac{1}{2}+r}{2}-\frac{3}{4}\right)\mathrm{e}^{-\pi kR} \quad \text{for } r>0.5\,. \end{aligned} \tag{A.8}$$

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